Variable Viscosity Effects Explained

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Once we are aware of the concept of dynamic viscosity and its temperature dependency, we can think of what does variable viscosity or more precisely, temperature-dependent viscosity do in a forced convection situation. For simplicity, we will ask this question in the context of a liquid flow through a parallel-plate channel as shown in Fig. 1.

Heating (or cooling) the liquid flow inside the channel makes the local temperature vary everywhere inside the channel. The temperature distribution of this process is got, in principle, by solving the general energy transport equation.

To solve the energy equation (that results in the temperature distribution knowledge inside the channel), *a priori* knowledge about the local velocity distribution is essential, which is got usually from the general momentum transport equation. While solving the momentum equation (for the convection process), we must in principle take into account the local viscosity variation, as it is a function of the local temperature. In essence, we are to solve a system of coupled partial differential equations.

Notice here, the word *coupled* represents a more intricate coupling than the already existing one due to the appearance of velocity in the energy equation. Local viscosity variation creates a two-way coupling because of which we cannot solve separately, either of the conservation statements while assuming it a constant (in the former case) allows us to solve the momentum equation separately. Figure 2 explains the situation with a collage.

To illustrate the relevance of the problem to engineering, we take a look at the specific case of the channel in Fig. 1. Treating the viscosity a constant everywhere inside the channel, the Navier-Stokes equation for maintaining a

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[1] intended as course notes.

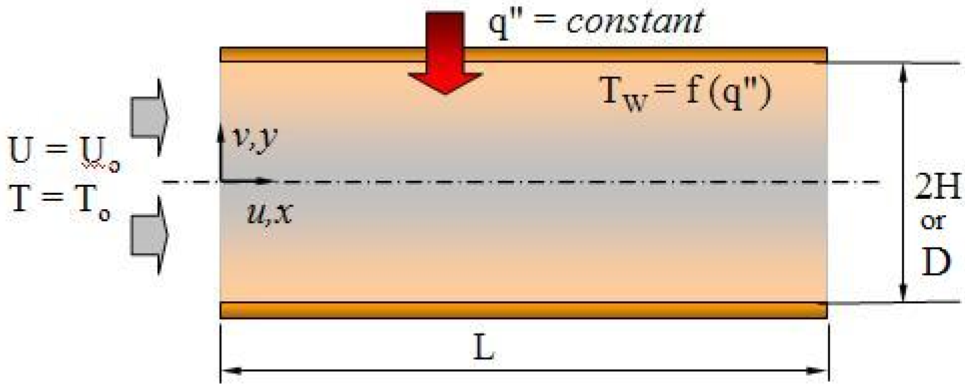

Fig. 1. Schematic of a parallel plate channel sustaining non-isothermal flow

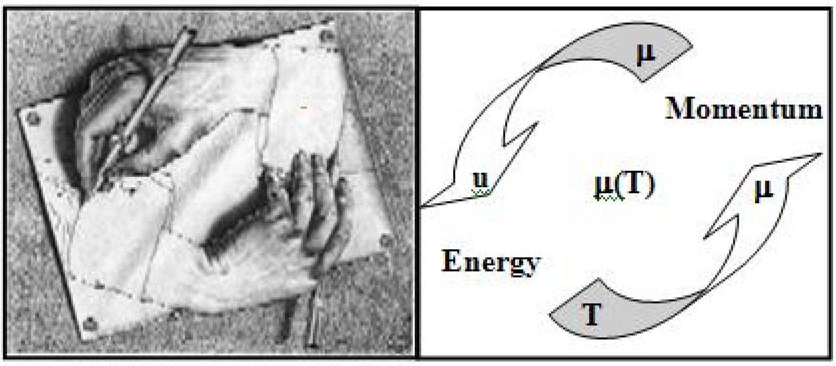

Fig. 2. a) Drawing Hands drawn in 1948 by Maurice Escher (Courtesy: Cordon Art B. V.-Baarn-the Netherlands) b) effect of temperature dependency of viscosity

hydro-dynamically fully developed laminar flow reduces to the algebraic form of the well known, Hagen-Poiseuille equation. Using this equation, for a given average velocity U, we predict the global longitudinal pressure-drop for the channel in Fig. 1 as

$$\Delta PL = \left(\frac{12\mu(T)}{(2H(or)D)^2}\right)U \tag{1}$$

This equation represents the balance between the longitudinal pressure force that has to be imposed to overcome the net friction force across the channel length (exerted by the channel wall on the flowing fluid) to maintain a flow with average velocity U. This pressure-drop is very important because that value determines how much pump power one would require for pumping the fluid across the channel length. This translates to the expenditure in electricity (usually) to drive the pump. Less the pressure-drop, less is the cost incurred.

For instance, for liquids, since the viscosity decreases with increase in temperature, the hotter the channel is, the 'easier' the resulting flow; in other words, for a mass flow rate to cross the length of the channel, the pressure-drop (hence the pumping power) required would be lesser than the corresponding case of constant and uniform viscosity (isothermal channel flow).

When the channel in Fig. 1 is isothermal (not heated or cooled), the viscosity in Eq. (1) is evaluated at the inlet (reference) fluid temperature. For non-isothermal flows (channel is heated or cooled), since the local viscosity is spatially varying, one of the following averaging options is usually adopted to find a suitable viscosity.

But how to predict this pressure-drop for non-isothermal channel flows? The first option is to evaluate the viscosity of Eq. (1) at the simple arithmetic average of the bulk temperature, defined as

$$T_b(x) = \frac{1}{2HU} \int_0^{2H} (uT)_x dy \tag{2}$$

evaluated at the entry ($x = 0$) and exit ($x = L$) of the channel of Fig. 1.

The second option is to evaluate the viscosity at the log mean temperature difference given by

$$T_{LMD} = \frac{T_b(L) - T_b(0)}{ln\left[\frac{T_b(L)}{T_b(0)}\right]} \tag{3}$$

and the third is to calculate an average viscosity along the entire channel, using the known temperature dependency $\mu(T)$ in terms of the bulk temperature, i.e., $\mu(T_b)$, hence

$$\mu = \frac{1}{L} \int_0^{L} \mu(T_b(x)) dx \tag{4}$$

and write Eq. (1) as

$$\Delta PL = \left(\frac{12\bar{\mu}}{(2H(or)D)^2}\right) U \tag{5}$$

Observe all of these options rely on bulk temperature. The bulk temperature, Eq. (2), in turn requires the prior knowledge of the velocity profile, which is influenced by the local viscosity!

This circularity is ironed-off by using the viscosity found from the averaging options, for both the differential (N-S equation) and algebraic (Eq. (1), in our case) momentum conservation statements. This makes the viscosity uniform and constant everywhere inside the channel and in principle, assumes the problem away.

However, all of these procedures for evaluating the viscosity are valid only when the outcome compares well with experimental results. Even after using the 'averaged' viscosity, if the global pressure-drop predicted by Eq. (1) deviates largely from the experimental results, we could deduce that the averaging options have failed to capture entirely, the effects of local viscosity variation in the channel.

Obviously, for this situation, Eq. (1) in its present form, is rendered useless for the practising engineer and a suitable modification of it is sought.

A similar line of argument based for instance on temperature dependent thermal conductivity, can question the use of the heat transfer solutions based on constant property assumption. Extensive investigations on the hydrodynamic and thermal (heat transfer) implications of variable properties in clear (of porous medium) fluid convection have been conducted for the past several decades.