Pipe Turbulence

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Everyday pipe flow is an easy example to observe Turbulence. Between 1883 and 1892, Osborne Reynolds performed experiments with water flow inside pipes and showed the laminar flow with parabolic velocity profile doesn't persist for higher flow rates. By 2006 we have experimental evidence that pipe flow turbulence may not persist forever. If we wait long enough, the flow could re-laminarize.

In the pipe flow experiments of Osborne Reynolds, the flow is continuously subjected to perturbations (small disturbances) arising from the inlet or from the roughness in the pipe inner wall surfaces or from the pipe vibrations. These disturbances try to unsettle the existing laminar flow at a certain flow rate. But the viscosity of the fluid damps out these disturbances as they are convected downstream from the inlet. So, the flow remains laminar at a long distance from the inlet. Beyond a certain flow rate, the perturbations don't decay but get amplified. This is known as a flow instability. In the pipe flow, this instability causes the laminar flow to change into a turbulent flow.

The schematic on the bottom in the above picture illustrates the results of Reynolds' dye streak experiments [1]. The first picture (top) depicts laminar flow and (middle) its transition to (bottom) turbulent flow. Turbulent flow is three dimensional (at each point parameters such as velocity can change in all three directions), neither steady nor parabolic velocity profiled. Reynolds was able to show from the results of his experiments that the parameter that controls the way pipe flow turns turbulent is the non-dimensional group

$$Re = \frac{\rho D U}{\mu} \tag{1}$$

[1] intended as course notes.

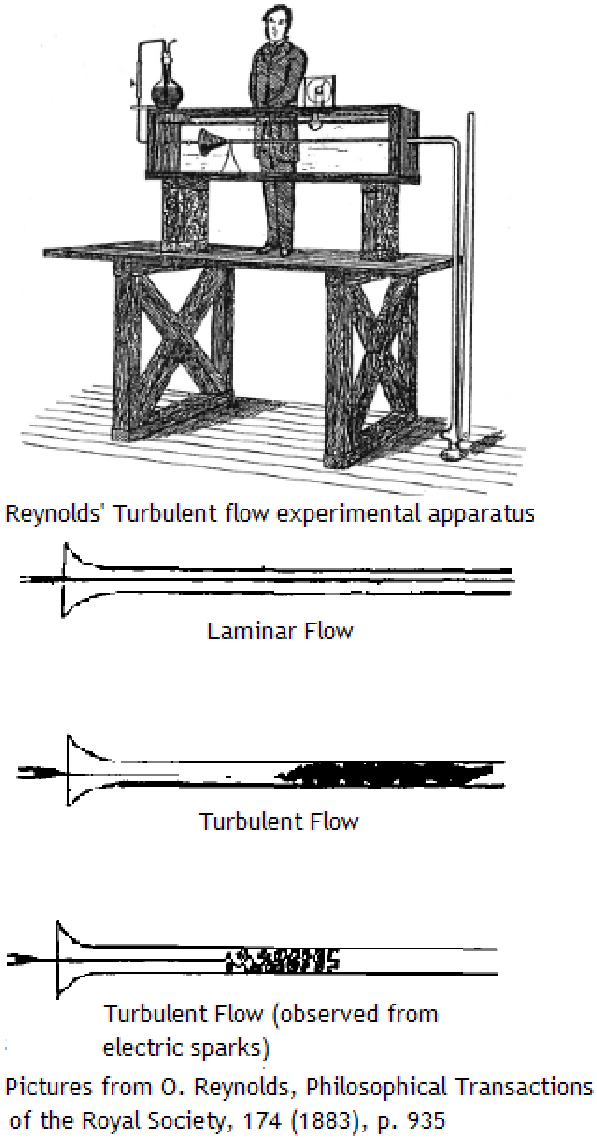

Fig. 1.

For a known pipe (inside) diameter, if we measure the average velocity of water flow inside the pipe, we can deduce the Reynolds number, once we substitute for the kinematic viscosity value from data book. Based on the Re, fluid flowing in a pipe - such as water running in household pipes - has a predictable way to transit into turbulence. At 20 degree C, the kinematic viscosity $latexnu$ for air is about $1.5e^{-}5m^2/s$ and for water is about $1.5e^{-}6m^2/s$. Hence, inside the same pipe, a water flow transits to turbulence at ten times lesser velocity than the corresponding air flow.

For infinitesimal perturbations, it can be shown analytically that laminar flow is stable for any Re value. Experimental research in recent decades have shown that if the perturbation magnitude is very small, laminar flow is stable even when $Re > 2000$. Laminar flow becomes sensible to perturbations beyond

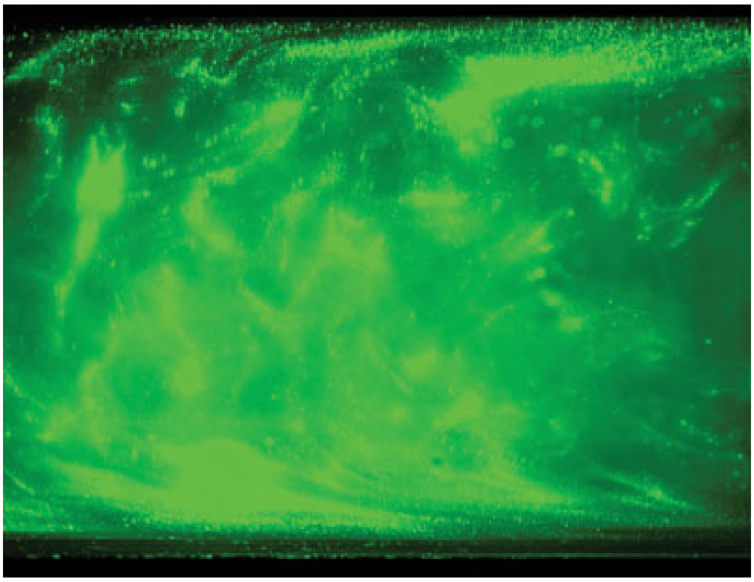

Fig. 2. Source: http://www.nature.com/nature/journal/v443/n7107/fig_tab/443036a_F1.html – see ref. [5]

Re > 2000 in general. But careful experiments suppressing the flow perturbations have recorded laminar to turbulent transition to happen only at a *Re* value an order above 2000. Such high *Re* flows are very sensitive to even small perturbations (a sneeze and a shake will do) and eventually becomes turbulent anyway. This shows transition instability depends on the amplitude and nature of flow perturbation.

The above picture visualizes pipe turbulence at Re = 5,000 using microscopic crystalline platelets illuminated with a sheet of laser light. These platelets align with shear flows, and changes seen across the flow indicate turbulent fluctuations.

In general, at relatively low flow rates, the resulting turbulence is transient. It can die out. Recent experimental results published in the Journal of Fluid Mechanics [2] and Physical Review Letters [3] have shown it is characterized by an exponential distribution of lifetimes. Turbulence set in at relatively low flow rates can decay and the flow can re-laminarize.

Based on their results, the above papers also suggested that beyond a critical Reynolds number (of 2300) the turbulence lifetime suddenly becomes infinitely large. That is, turbulence can persist indefinitely beyond a critical Reynolds number. But this view is challenged recently by Hof and collaborators [4] who reported their experimental and simulation findings in 2006 in Nature.

The details of their beautiful experiment and analysis require a detailed note. Briefly, their results doesn't indicate an infinite lifetime for turbulence in pipe flows beyond a critical Reynolds number. Only the finite lifetime for turbulence (with exponential increase in lifetime with increased Re) is predicted by their results. In pipe shear flows, all turbulence should decay eventually and the flow should relaminarize, if one waits long enough. Long enough is the key. Because, the rapid exponential increase of turbulence lifetimes with increasing Reynolds number makes it difficult to guess its transience. As Hof et al. suggest, this is perhaps why it has not been suspected or observed previously.

This point can be appreciated with the example that Hof et al. mention in the conclusions of their paper. To detect the decay of turbulence in a garden hose at a flow rate as low as 1 l/min (Re = 2,400) would require a physical length of the tube of 40,000 km, about the Earth's circumference, and an observation time of almost 5 years.

Here is another example given by Daniel Perry Lathrop in Nature [5]: For typical flows, say in a city water main of 60-centimetre diameter, extrapolating the result of Hof et al. gives a decay time of $10^{3,000}$ years for the turbulence to die out. A painfully slow decay, which, for all practical purposes, can be treated to persist indefinitely.

In practical situations, the irregular flow patterns of turbulence doesn't match up with the smooth analytical results of particular solutions of the Navier Stokes equations. It is believed for years that turbulence is a nonlinear solution of these equations; one step beyond chaos, but a persistent steady state. This thesis is what is now questioned by the results of Hof et al. [4] that show at least pipe flow turbulence instigated by a small disturbance may not be persistent (i.e. having a indefinite lifetime) even beyond a critical Re of transition. The flow returns to a laminar state if one waits long enough.

These new results imply among other things that in pipe flow the turbulent and laminar states remain dynamically connected. Control of such turbulence is a possibility. There is a chaos angle to all this, which require a separate note.

References

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