Turbulent Flow around Bodies

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We have discussed in earlier notes, what is Turbulence and a specific example of what is Pipe Turbulence. Fluid flow around solid bodies offer one more situation of laminar to turbulent flow instability.

Suppose we hold a long cylinder of circular cross section in a steady fluid flow such that the flow is perpendicular to the axis (lengthwise direction) of the cylinder. Dipping a straw in a calmly moving river or backyard stream is an example. Wading a pencil or our finger in a stagnant pool or a filled bath tub is an equivalent. When does such a flow obstructed by a solid body of definite shape (here it is circular cross section) turn turbulent?

A measure to characterize such a flow is the non-dimensional Reynolds number (Re), defined as the product of the uniform undisturbed flow velocity far away from the cylinder and diameter of the cylinder divided by the kinematic viscosity of the fluid. For the moving finger in a stagnant pool, the velocity and diameter to find Re are that of the moving finger. We can study the configuration for increasing Re values. For flow around bodies the transition to turbulence is dramatic. Interesting intermediate steps exist.

For a slow flow with Re less than 40, the flow around the cylinder is almost symmetric about a plane along the axis parallel to the flow. The flow streamlines that separate at a distance upstream of the cylinder, meet symmetrically at a distance downstream of the cylinder. This is shown in the accompanied picture, simulated in our local computers by solving iteratively using computers, the discretized (algebraic) version of the Navier Stokes equations with proper boundary conditions. The color from red to blue indicate the decreasing magnitude of local velocity of the flow. For instance, upstream of the cylinder (black circle), along the axis of symmetry, as one would expect, the flow stops,

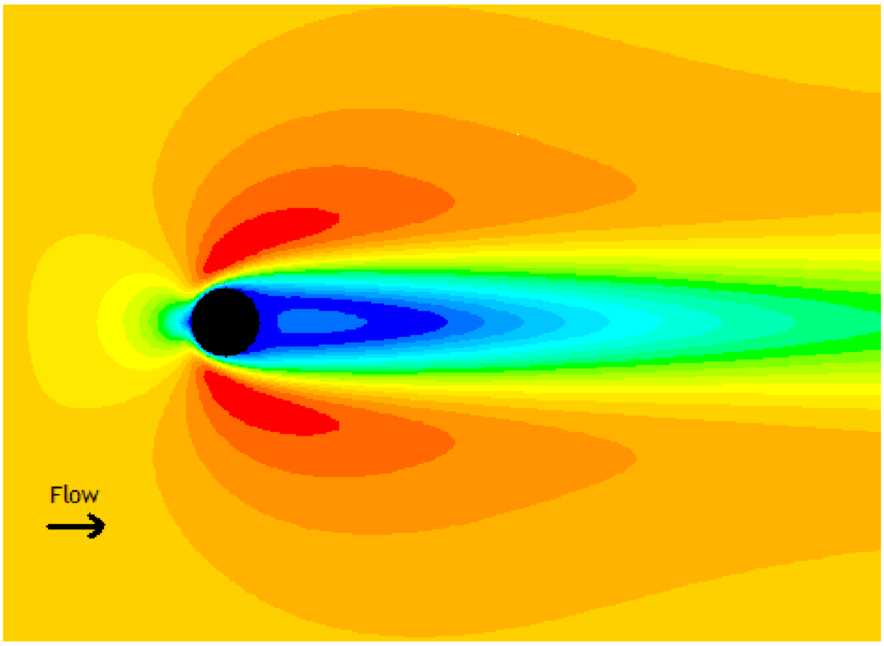


Fig. 1.

indicated by the blue color. This is called a stagnation point. The definition of this is a bit more involved, which we can skip in this discussion. Downstream, in the shadow region of the cylinder, again there is reduced flow.

By potential flow theory (developed by the likes of Euler (pronounced 'Oil'er not 'Yu'ler) and Daniel Bernoulli), in which the flow is treated inviscid (viscosity less), one wouldn't expect a pressure drop to exist across the cylinder in the flow direction. The streamlines should separate upstream of the cylinder and meet immediately after the cylinder downstream. There is no momentum diffusion for the flow to exist over and across the cylinder. If this is true, sailing ships doesn't require any power to wade the water, as the flow wouldn't 'know' or be influenced by the presence of the ship. But this is not the real case and there exists a pressure drop across the cylinder indicating a momentum diffusion. This situation is known as *d'Alembert's paradox*. Ludwig Prandtl successfully explained this.

Because of the finite viscosity of the fluid there exist a momentum diffusion in this flow past the cylinder, with the cylinder wall resisting and decelerating the nearby flow within a region called the boundary layer. Observe the color change from blue to almost red across a small distance above and below the cylinder. This results in a small pressure drop across the cylinder in the flow direction. The rest of the flow region away from the cylinder retains identical flow speed at all locations (almost of shades of orange and red color). However, the flow field everywhere in this situation is still laminar.

Source: `http://www.nonoscience.info/`

We now increase the Re to anywhere between 40 and 100 for the flow. In our initial example of a steady flowing river, we cannot do this by increasing the speed of the river. Alternately, we can increase Re by increasing the diameter of the straw. Instead of the straw, by dipping our leg into the river we could increase the Re about five times. But this ensures the Re to be between 40 and 100 only if the initial Re value (when the straw was used) is correspondingly five times lesser. In the other example, the straw or pencil just needs to be waded faster in the stagnant pool or bath tub. By keeping the Re between 40 and 100, we can observe now the region of flow near the cylinder surface begin to separate downstream from the top and bottom of the circular cross section. A periodic flow of undulating wakes can be seen. Two symmetrically placed recirculating eddies develop behind the cylinder and feed the wake of the cylinder with vorticity.

In the Pipe Turbulence case the instability caused the laminar flow to change its character in a step function to another qualitatively different transition flow. Here this is not so. Although the steady flow shown in the picture above is a valid solution even when Re is increased beyond 40, it doesn't persist because of susceptibility to even small perturbations. The laminar steady flow around the cylinder for $Re < 40$ as shown in the above figure, when the Re is increased, transforms into another kind of laminar flow, an unsteady one with periodic oscillatory wakes leaving from the back of the cylinder. Unlike the Pipe Turbulence case, this instability can be predicted from perturbation analysis by linearizing the Navier Stokes equations. Also, the instability doesn't depend on the amplitude of perturbation and so the critical Re value beyond which the instability sets can be analytically found.

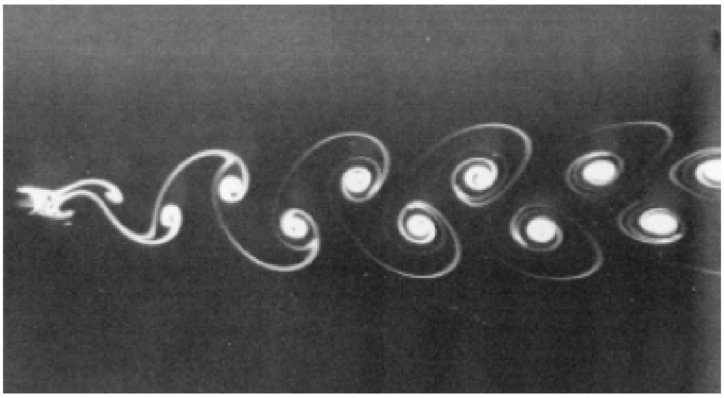
Increasing the Re further, for $Re > 100$, results in the unsteady separation of the viscous boundary layers that coat the cylinder surface, as discussed above. We can observe the time periodic alternating ejections of vortices from the top and bottom portion of the circular cross section of the cylinder. This beautiful phenomenon as seen in the picture below is known as the Karman vortex street, named after Theodre von Karman (first generation disciple of Ludwig Prandtl, mentioned above), who first explained their formation.

Here is a YouTube video of all of the phenomenon explained until now. Visit ZnahNah's YouTube page to view other such simulations.

`http://www.youtube.com/?v=qpDKRrS9aqE`

Observe initially how the flow around the cylinder is almost symmetric when the Re is low and as Re increases, the subsequent laminar boundary layer separation into the alternate instances of vortices being generated at the cylinder downstream section.

The flow in the above video is not real but again a computer simulation done



Karman vortex street behind a cylinder placed in uniform flow. Re ~ 300 [Courtesy: Sadotoshi Taneda; from An Album of Fluid Motion by Van Dyke (1982)]

Fig. 2.

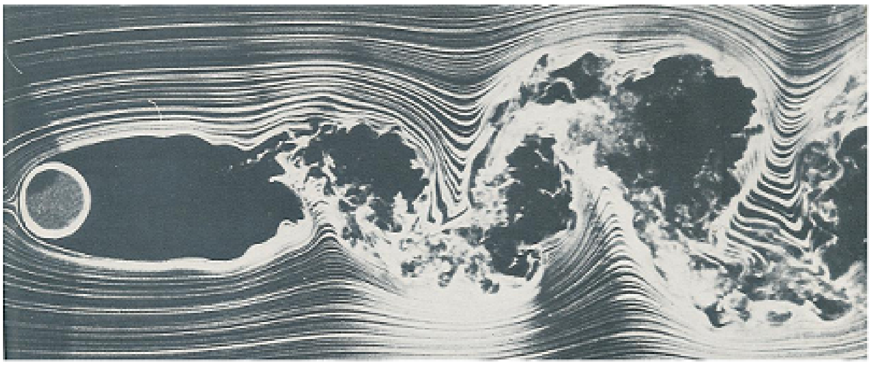
by treating the fluid flow as an equivalent of a gas that can travel in discrete locations on a lattice, with certain rules of travel governed by the Boltzmann equation. The technique is called the ¡a Lattice Boltzmann method. More on this another time.

As the Re is increased further, turbulence is seen to set in at the far wake, a result of the instability in the Karman vortices. For further increase in Re, the turbulence reaches the cylinder from downstream and when the Re reaches in thousands the entire wake behind the cylinder is turbulent. The flow is essentially aperiodic but still retains remnants of the periodic vortices.

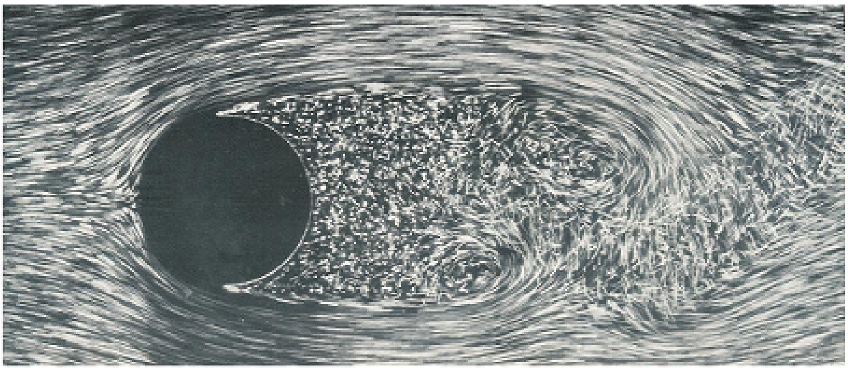
At higher Re the flow inside the wake becomes turbulent but the mean flow around the body and wake is still laminar as seen in the picture below. A region of fine grained turbulence is seen immediately next to the back of the cylinder as shown in the second figure below. Further downstream a more organized spatially larger vortices exist. Interestingly, outside the vortex, the flow remains laminar. This shows both laminar and turbulent flow can coexist and even spatially alternate in this configuration as shown in the above picture.

One more thing. The boundary layer attached to the cylinder that we kept talking about also remains laminar while all of these instability triggered phenomenon happens downstream. Around a Re between $latex10^5$ and $latex10^6$, this laminar boundary layer itself, instead of shedding itself into the cylinder wake, turns turbulent and attaches itself to the cylinder. Eventually, for higher Re, this too separates. As the Re is increased further, this boundary layer transition from laminar to turbulent happens further and further upstream in the cylinder.

To summarize, flow around bodies - we considered only a cylinder here - exhibit transition to turbulence in a markedly different way from that observed in Pipe



(a) Visualizing turbulent cylinder wake at Re = 10000
[Courtesy: Thomas Corke and Hassan Nagib; *from An Album of Fluid Motion by van Dyke (1982)*]



(b) a closer look at Re = 2000 - patterns are identical as in (a)
[Courtesy: ONERA pic. Werle & Gallon (1972) *from An Album of Fluid Motion by van Dyke (1982)*]

Fig. 3.

Turbulence, where it is a step change from laminar flow to transition beyond a critical Reynolds number. In flow around bodies, the transition to turbulence happens in steps; laminar, unsteady laminar, transition, turbulence. Further, the turbulence sets in the far wake, far downstream of the body (cylinder), and gradually approaches the cylinder and further upstream, for increase in the Reynolds number. The phenomenon extends to Re running in thousands.

Let me stop the flow here. Traverse a pencil held vertical in a stationary pool of water. For enough speed of traverse you can spin off the vortices that we discussed here.

References

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- Turbulent Flows, Fundamental Experiments and Modelling edited by G. Biswas and V. Eswaran, Narosa Pub., 2002 - Narosa Link
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