# Turbulence

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Let me not begin with what Werner Heisenberg [1 - see endnotes] and Sir Horace Lamb [2] wished to ask God. Instead we observe across centuries what Leonardo da Vinci has painted on everyday turbulence gushing out from a duct.

Everyday observations allow us to ¡em¿describe¡/em¿ turbulence loosely as a rushed, gushed, mixed, swirling, twirling, frothing, bubbling, ”confused” flow. That description would classify as turbulent the rain water celerities on the road, the hot coffee jet traveling from one cup to another, the cold air that coats and flows around our naked face while traveling in a bike, the smoke from the factory chimney, the clouds and ocean currents, the atmosphere of the Suns and their planets, the blood flow in our arteries - the list is endless.

Science is less romantic but better in description with a systematic study.

Water flow inside a pipe, as observed by Leonardo, is a daily affair. At a sedate pace (determined by the pipe and fluid configuration) such internal pipe flows are termed laminar - modelled as if, laminae of fluid sliding over each other. Fully developed laminar flow inside pipes with circular cross section exhibit a parabolic velocity distribution with zero at the walls and maximum velocity at the axis. Fully developed flow means, the flow retains its radial velocity profile while flowing along the length of the pipe. This render the longitudinal pressure drop linear. This parabolic velocity profile of flow was analytically deduced and experimentally observed independently by Hagen (1839) and Poiseuille (1841). Given an infinite length of the pipe, such a parabolic profiled laminar flow of any fluid is inevitable inside a pipe. The analytical solution could be vindicated.

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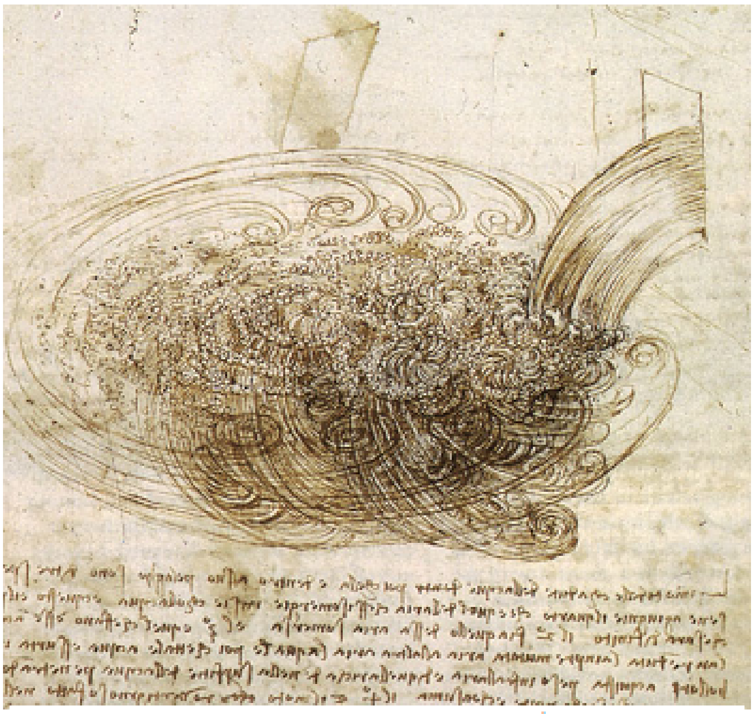


Leonardo's imagery of falling water (1508-09)
[Source: http://www.visi.com/~reuteler/leonardo.html]

Fig. 1.

But laminar flow runs inside the beautiful confines of analytical viscous fluid mechanics, useful to model flows in carefully constructed laboratory experiments by hardworking fluid mechanist. When they step out of work, turbulent cold air gushes in through the opened door to upset their notes of laminar analytical labour. Unlike what our introductory fluid mechanics books suggest, turbulence is the norm, not an exception like laminar flow.

Turbulent flow results when the instabilities in a flow are not damped persistently by the viscous action. The ensuing turbulence in the flow exhibit rapidly varying velocity fluctuations with respect to space and time. If we plot the measured local velocity of a fluid flow as a continuous function of time, for a laminar flow that curve is smooth. The curve develops many random kinks and wiggles when the flow is turbulent as can be seen in the picture below. Similar is the velocity plot when done in space in many location of the flow at a particular instant. The plot resembles the graphed fluctuating fortunes of the SENSEX (SENSitive indEX of the Bombay stock exchange) or the hackneyed ECG of a person about to die in the Indian movies.

Intriguingly, when the kinks are resolved in higher magnification (i.e. plotting measurements done at shorter time or spatial intervals), we see more kinks of smaller scale. Analogously the SENSEX graph shows similar kinky behavior when done for a day (shorter time) or for several months (longer time) as shown

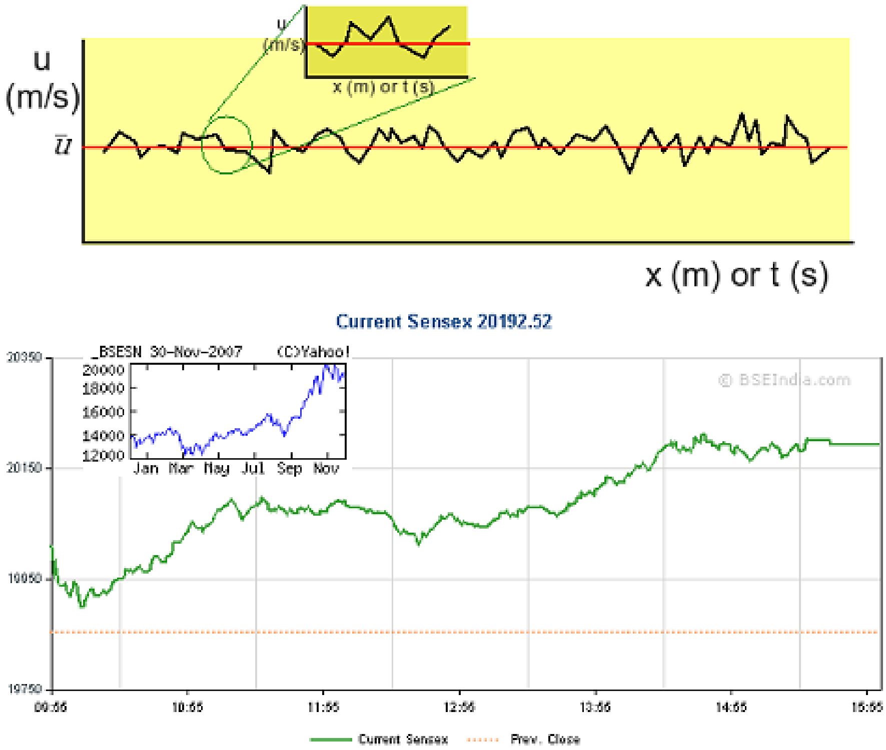



Fig. 2.

in the inset. This behavior of velocity fluctuations doesn't extend endlessly for all further finer scales. Before we jump to spot a fractal, at very high resolution a smooth function for velocity versus time or space is reached.

The physical nature of turbulence is understood using the concept of an eddy. In fluid mechanics, an eddy (or a vortex) is understood to be a macroscopic fluid element in which the microscopic elements composing the eddy behave in some ways as a unit. Macroscopic means a region of fluid and microscopic means the smaller sub-regions within the macroscopic fluid region considered. A turbulent flow is understood to comprise many such eddies (or vortices) of different sizes. Groups of smaller sized eddies of differing behaviour can congregate within a larger eddy. Fully turbulent flow means eddies of all shapes and sizes exist.

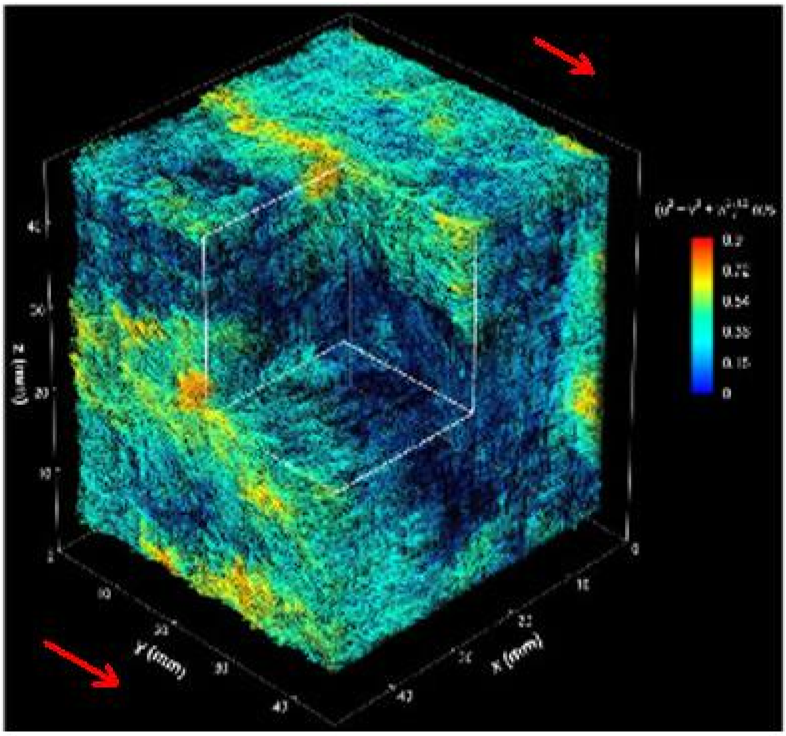
These eddies are continually formed in fully turbulent flow by drawing on the kinetic energy from the mean flow. The large eddies also keep breaking down into smaller ones, which break down further into smaller ones, and so on. One of the features of a turbulent flow that accompanies this cascade of eddies is the energy cascading or the transfer of energy in an average sense from large scale eddies to progressively smaller scales. At small enough sizes, the eddies die out by dissipating their kinetic energy as heat through local viscous action of the fluid. The British meteorologist Lewis F. Richardson described this process in a now famous verse:

> Big whorls have little whorls, Which feed on their velocity, And little whorls have lesser whorls, And so on to viscosity.

Turbulent flow is intrinsically three dimensional and random in space and time. Observe the experimental measurement of instantaneous velocity distribution in a turbulent flow field inside a square cross sectioned duct in the accompanying picture below. These data were recorded using Holopgraphic Particle Image Velocimetry systems developed in the Laboratory for Experimental Fluid Dynamics by Prof. J. Katz and co-workers. For clarity, the experimenters [3] have subtracted the center-line mean velocity, $U = 2.1m/s$, from each vector. The colors indicate the magnitude of the velocity (from red high to blue low) and the red arrow indicates the direction of the flow. In a sense, over the centuries,we are now in a position to do better descriptive drawings of turbulence when compared to the drawing of Leonardo in the first picture. However, no two experiments reproduce exactly all the details of a particular turbulent flow.

This is because of the sensitive dependence of the flow to small disturbances in the initial and boundary conditions, which an experimentalist cannot control with any required precision. The randomness has prompted researchers to wonder whether it makes sense to try for a precise mathematical analysis of such flows. In theoretical analysis one assumes a multitude of almost identical experiments to have been performed and deals with the "ensemble" of collected data of the measured parameters. Averaging data from several (in principle, infinite) such ensemble realizations leads to the statistical description of turbulent flows. The mean flow in the above second picture [tex]baru[/tex] is understood to have been arrived at by averaging the measurements over a large number of almost similar experiments in a turbulent flow configuration. The fluctuations (the kinks in the curve) are understood to be the variations from the mean caused by the turbulence itself [4]. For laminar flow, these fluctuations die out considerably.

The advantage of the above approach is statistics of flow is reproducible, while the result from an individual experiment (contained as a data point in the statistics) is not. The turbulent flow field, when analyzed in a length scale that is smaller than the length scale of the flow configuration but big enough when compared to the length scale of the smallest eddy in that flow, exhibit a statistically uniform behaviour. However, the basic dynamical equations, the Navier and Stokes equations [5] of momentum conservation for a fluid flow, should hold unequivocally for both laminar and turbulent flows. This means turbulence is a consequence of these equations rather than a breakdown of such a momentum conservation model. Compared to the statistical description from experiments then, the solution of Navier Stokes equations suggest possibility of predicting the results from the individual instances of experiment. A caveat is of course, the general solution for the Navier Stokes equations is yet to be



Instantaneous three dimensional velocity distribution of turbulent flow in a square cross-sectioned duct measured using holographic particle image velocimetry (PIV). Color scales indicate velocity magnitude with blue as low and red as high values. Flow is along the arrow marked direction.

Image source:
[http://www.me.jhu.edu/%7Emeneveau/gallery.html]

Fig. 3.

found. Even the existence of a solution, barring some brave attempts [6], is yet to be proved. This is where fast computers come in.

In computational fluid dynamics (CFD) turbulent flow researchers usually solve the Navier Stokes equations in a particular turbulent flow configuration and from the individual results, calculate the statistical properties of the flow enabling comparison with the statistical results of analogous experiments. Another theoretical approach is to work straight with the statistical equations, with suitable hypotheses or "closure models" about the involved unknowns. I am not an expert to write more about this approach.

Some of the physical features of turbulent flows are their inherent randomness and three dimensionality, comprising of features of different scales (like eddies of many sizes), random variations of vorticity in the small scale and their nature to dissipate energy. However, turbulence is essentially a continuum phenomenon (one need not describe turbulence using fluid molecules). Major applications of turbulence hinges on the fact that it enhances the dispersal and transport of material, momentum and energy when compared to simple diffusion or laminar flows. More on each of these in follow-up essays. This is as far I can go without equations.

At the start, we refrained from the customary quoting of scientists on how difficult turbulence is. Citing the frustrations of some of the giants of Science is not the best way to encourage young researchers to pursue the field of turbulence. To illuminate this point I quote here from the starting paragraphs of an excellent recent review article [7]

> One need not apologize for or despair over the difficulty of turbulence. For one thing, turbulence has contributed several ideas and tools of lasting value to neighbouring areas of physics. A sampling includes negative temperature, anomalous diffusion, and the concept of power-law scaling in many-body problems. The powerful notions of scale invariance and universality were first proposed in the context of turbulence. In general, turbulence is a playground for solutions that are non-unique or that depend sensitively on initial conditions [...]
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> [...] turbulence is not a single problem but rather a huge field with pivotal applications in engineering, geophysics, astrophysics, and cosmology. It is also an excellent source of problems of pure mathematics. Applied mathematicians, meteorologists, and engineers often focus on particulars like drag and pressure drop, mean velocity distributions, mixing efficiencies, and dispersion rates. Indeed, by considering the totality as "the problem of turbulence," one can justify the claim that the problem remains unsolved. In like manner, the insistence on a complete first-principles understanding of the structure of complex atoms and molecules might lead to the conclusion that quantum mechanics is an unsolved problem.

Along with the above encouragement, to convey the uncertainties that charm the field of turbulence, let us end this essay with a quirky quote alluded to a noted expert [8] on turbulence. To a question that went something like "How to recognize a flow is turbulent?", his answer was

> Turbulence is like pornography. You will know it when you see it.

## References and End Notes

(1) "When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." said Werner Heisenberg once. Source: Dr. Brian Thurow `http://www.eng.auburn.edu/users/thurobs/Turb.html`.

(2) A possible derivative of the above quote is ascribed to Sir Horace Lamb,

author of the seminal Hydrodynamics, who in 1932 while addressing the the British Association for the Advancement of Science said, "I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic." I read this in Prof. Parviz Moin's Scientific American Article `http://turb.seas.ucla.edu/~jkim/sciam/turbulence.html`, a good, equation-free, general introduction to how computers are used in tackling turbulence.

(3) PIV image is from the paper Tao, B., Katz, J., and Meneveau, C. (2000). Geometry and scale relationships in high Reynolds number turbulence determined from three-dimensional holographic velocimetry Physics of Fluids, 12(5) DOI: 10.1063/1.870348 and pdf of the article from author's website.

(4) this is discussed in the 1st Chapter of An Introduction to Turbulent Flow by Jean Mathieu and Julian Scott (2000, Cambridge Uty. Press), a good book that I am using to learn some of the key ideas in turbulence.

(5) I am deliberately keeping this post equation free. Read the Wikipedia page for Navier Stokes equations for an introduction or check `http://www.navierstokes.net/` just for the equations and a feline perspective.

(6) If you can handle comfortably some mathematical language, read the excellent post titled Why Global Regularity for Navier Stokes is Hard by Terrence Tao at `http://terrytao.wordpress.com/`.

(7) Falkovich, Gregory and Sreenivasan, Katepalli R. "Lessons from hydrodynamic turbulence," Physics Today, vol. 59, no. 4, pages 43-49 (April 2006). This article is a good review of where the field is. Some sections of the review is accessible to all of us.

(8) From academic discussions (i.e. college gossip) I know the name of this scientist but couldn't authenticate hence refrain mention.