Convection Carnot Engine

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Convection can be distinguished into two types: forced and natural or free [1]. When thermal gradients cause a density gradient that result in local bulk motion in a fluid due to buoyancy forces, natural convection is realized [2]. The natural convection phenomenon can be thought as a Heat Engine [3]. A heat engine is a device that performs the conversion of heat energy to mechanical work by exploiting the temperature gradient between a hot source and a cold sink.

The analogy between natural convection and heat engine is explained in the figure below. Heat engines run with a working fluid that undergo a thermodynamic cycle repeatedly to generate power. A thermodynamic cycle is a closed set of linked processes, when executed once, changes the thermodynamic state of a system (working fluid) through change in properties like pressure and temperature, bringing the system back to its original state. The conversion of heat (from, say, fuel) to work (that we use) happens in this cycle.

Natural convection can be thought of as a continuous thermodynamic cycle undergone by the convecting (working) fluid. Let us see how. In an ideal situation, the processes that constitute the natural convection cycle inside an enclosure are as follows:

(1) Process 1: Here the fluid packet considered to experience the force imbalance is composed of mass say dm. The actual movement of this packet is because it receives heat energy at the bottom from a heat source isothermally because of which it immediately expands. In effect, it receives heat energy by undergoing an isothermal expansion process.
(2) Process 2: With this energy, it increases its volume and as seen earlier experiences a force imbalance resulting in the displacement, which causes further expansion as, the packet raises. This results in an adiabatic ex-

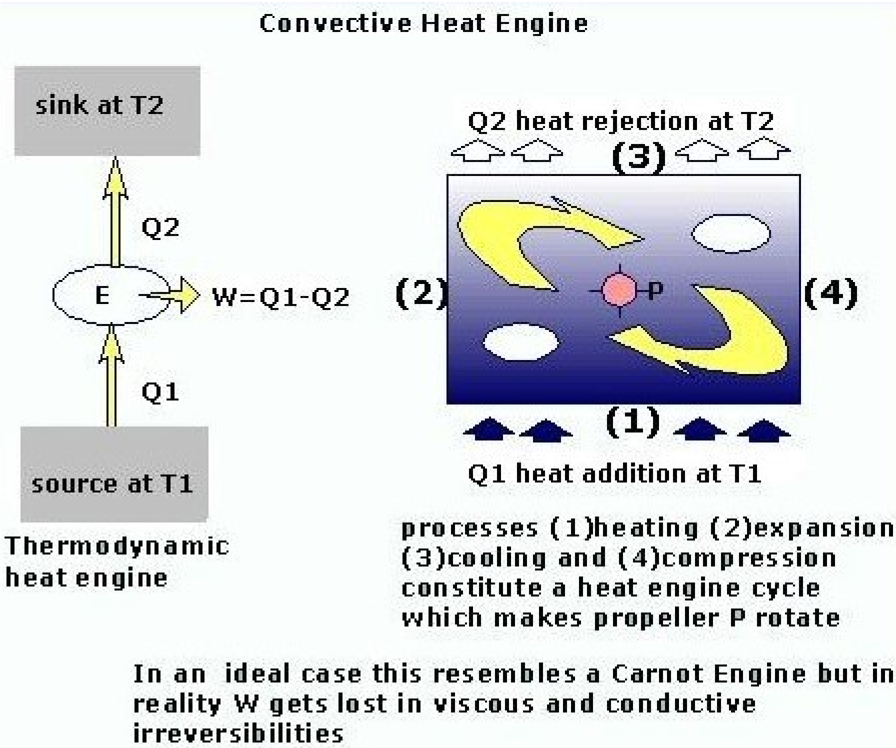

Fig. 1. Convection Carnot Engine Analogue

pansion process in which the packet moving up can perform some work.

(3) Process 3: As it goes up the packet loses the remaining heat energy to the surrounding at the top to maintain equilibrium. This it does as an isothermal compression process as it cools and contracts at the top.
(4) Process 4: Finally this cooled packet is what we get circulated back to the bottom, if we have to account for the lost mass in the bottom by mass conservation principle. This happens through an adiabatic compression process, as the packet further contracts as it comes down.

The fluid packet executes a cycle comprising heating - expansion - cooling - compression the processes that constitute a Carnot Heat Engine Cycle [4]. This heat engine cycle was an idea proposed by Nicolas Leonard Sadi Carnot [5] in 1824. We use this ideal cycle today as a reference theoretical maximum efficiency cycle for evaluating any real heat engine thermodynamic cycle like the Otto cycle or Diesel cycle.

Existence of a work producing potential in the Natural Convection flow inside enclosures can be made evident by inserting a propeller made of a light material across the flow path of the fluid packet. The circulating convection wheel in water can be visualized in the laboratory with strewn aluminum powder. In reality, the work output of this cycle, unfortunately, is sufficient only to accelerate the fluid packet against the viscous drag - the fluid brake opposing motion - it experiences on the way.

We could quantify this idea using simple scaling equations. The buoyancy force generated in the above enclosure convection system is the chief cause for the work producing potential of the thermodynamic cycle.

As an order of magnitude, this work done by the convecting system as a result of the buoyancy force is given by

$$W \sim (\rho U \Delta T \cdot g \cdot \alpha) \tag{1}$$

where ρ is the density and $latex\alpha$ is the thermal diffusivity of the convection fluid, U is the velocity of the convection wheel, $\Delta T = (T_1 - T_2)$ is the temperature difference across the enclosure as defined in Fig. 1 and g is the acceleration due to gravity.

The heat energy added to the convection system, again as an order of magnitude, is given by

$$Q_1 \sim (\rho \cdot c_P \cdot U \cdot \Delta T)/L \tag{2}$$

where c_P is the specific heat of the convecting fluid at constant pressure.

The efficiency of the Convection Carnot cycle can be defined as the ratio of how much useful work one derives from the rotating convection wheel (given in Eq. 1) and how much heat energy one spends to achieve this (given in Eq. 2). This ratio is given as

$$\frac{W}{Q_1} \sim \frac{gL\alpha}{c_P} \tag{3}$$

The notation on the LHS of Eq. (3) corresponds to conventional thermodynamic usage (see Figure 1).

Typical to the convection system described in Fig. 1, the $W \ll Q$ and $Q1 \sim Q2$, which results in the low efficiency. The fluid packet received some energy in the enclosure bottom, which is heated. Subsequently, this energy will be dispersed from it into the surrounding fluid, depending on the strength of c_P and α. Before all the energy from the packet diffuses, the fluid packet raises because of buoyancy. This displacement can lift a weight, i.e. do work. However, the displacement has to overcome the viscous resistance and the thermal diffusion along the way for which some of the energy is used. The remaining energy in the packet can be realized as work output.

The convection situation described obeys the First Law of Thermodynamics. However, after overcoming the fluid viscous dissipative brake and thermal diffusion, the remaining kinetic energy in the packet is almost a small perturbation in the First Law of Thermodynamics. When Convection is modelled as a Carnot engine, it should be completely reversible.

From the Second Law of Thermodynamics one could infer the change in entropy for such an ideal convection cycle is zero. In reality, the viscous dissipation and diffusion/conduction irreversibility generates enough entropy to restrict the work potential of the convection heat engine. This is embedded in the construction of the dimensionless group called the Rayleigh Number.

A recent research paper by Prof. V. A. F. Costa [6], explores nicely, the role of viscous dissipation in natural convection engine discussed here. Additional references that explain the convection process are provided.

Reference

Narasimhan, A., "Convective Carnot Engine," Phys. Educ. **35** (2000) (3), pp. 178-181, DOI 10.1088/0031-9120/35/3/307 [Link to Abstract-INSPEC]

Further Reading

(1) Convection that cannot be identified distinctly as either forced or free is mixed convection.
(2) Read 1) Free Convection for Dummies [Link] and 2) Free Convection and Rayleigh Number [Link] for an explanation of natural convection phenomenon.
(3) Heat Engine Link [`http://en.wikipedia.org/wiki/Heat_engine`]
(4) Carnot Cycle Link [`http://en.wikipedia.org/wiki/Carnot_cycle`]
(5) Sadi Carnot [Wikipedia Link]
(6) On natural convection in enclosures filled with fluid-saturated porous media including viscous dissipation, V. A. F. Costa, International Journal of Heat and Mass Transfer, Volume 49, Issues 13-14, , July 2006, Pages 2215-2226 [Link to abstract]
(7) Physical Fluid Dynamics, D. J. Tritton, (1988), Oxford Science Pub. Amazon Link
(8) Convection Heat Transfer, 3rd ed., A. Bejan, (2004), John Wiley and Sons Amazon Link
(9) Convection - M. G. Velarde and C. Normand, (1980), Scientific American
(10) Rayleigh Benard Convection: Structures and Dynamics - A. V. Getling, (1998), World Scientific Amazon Link