# Heatlines

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Streamlines visualize two dimensional steady fluid flow. They are the pictorial representation of the stream function defined using the equations

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{1}$$

such that the mass conservation is satisfied as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

Here $u$ and $v$ are the fluid velocity components in the Cartesian $x$ and $y$ direction and $\psi$ is the stream function.

From the stream function definition it is evident that no flow happens orthogonal to the streamline ($\psi$ = constant lines). A plot of the stream function $latex\psi$ from its minimum to maximum value visualize the pattern of steady fluid flow.

A Heatline is similar to a streamline, but visualize net energy flow in a convection or conduction heat transfer situation. The energy conservation equation for an incompressible steady flow with zero heat generation and negligible viscous dissipation can be written in two dimension in Cartesian form as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

Here $\alpha$ is the thermal diffusivity of the fluid in $m^2/s$ and $T$ is the temperature.

Rearranging Eq. (3) as

$$\frac{\partial}{\partial x}\left(\rho c_P u T - k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho c_P v T - k\frac{\partial T}{\partial y}\right) = 0 \tag{4}$$

one could define a function H from Eq. (4) such that

$$\partial H \partial y = \left(\rho c_P u(T - T_{ref}) - k\frac{\partial T}{\partial x}\right) \tag{5}$$

and

$$\frac{\partial H}{\partial x} = \left(\rho c_P v(T - T_{ref}) - k\frac{\partial T}{\partial y}\right) \tag{6}$$

Equation (5) represents the net energy flow by convection and conduction in x direction, while Eq. (6) represents the same for y direction. Akin to the stream function formulation, across each $H$ = constant line, the net flow of energy by conduction and convection is zero. The difference between two successive heatline values (say, $H_1$ and $H_2$) should equal the total energy flow rate per depth of separation of the heatlines.

The $T_{ref}$ in Eq. (5) and (6) is an arbitrary constant. By convention (and by necessity when heat generation is involved), it is taken as the lowest temperature in the heat transfer configuration.

The concept of Heatline for visualizing convection is introduced in 1983 by Kimura and Bejan. Figure  is an example of heatlines visualizing natural convection flow of water inside a square enclosure, reproduced from their journal paper.

Flexing our laptops, given in Fig.  are heatlines of natural convection flow of water inside a square enclosure stuffed with a porous medium. The solid matrix of the porous medium for the example below can be thought of as uncompressed wire crimps of permeability about $0.01m^2$, contained in a box of length $10cm$.

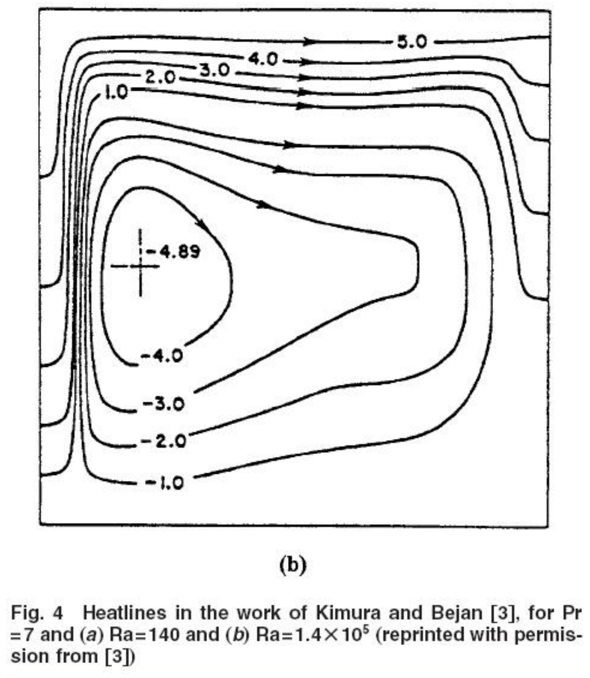


Fig. 1. Heatlines. Source – Reference [1]

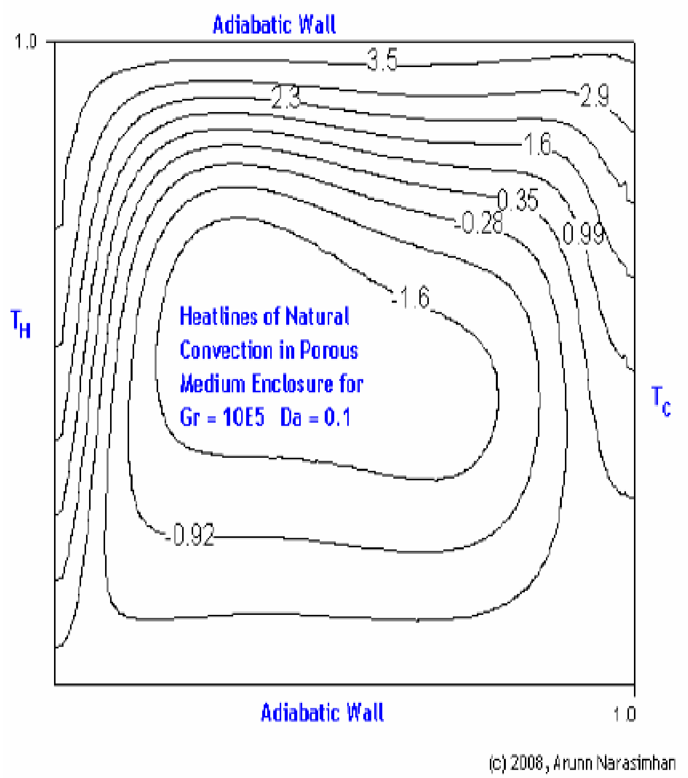


Fig. 2. Heatlines of natural convection inside a porous enclosure



Source: `http://www.nonoscience.info/`

Observe that the heatlines are not perpendicular to the isotherms indicating the presence of convection (instead of pure conduction).

### Further Reading


(1) Kimura, S., and Bejan, A., 1983, The Heatline Visualization of Convective Heat Transfer, ASME J. Heat Transfer, 105, pp. 916919.

(2) Costa, V. A. F., 1999, Unification of the Streamline, Heatline and Massline Methods for the Visualization of Two-Dimensional Transport Phenomena, Int. J. Heat Mass Transfer (doi: 10.1016/S0017-9310(98)00138-0)