

How to quickly cool a bottle of drink

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Say, you have your favourite drink in a bottle (water, in my case, beer for most of my friends and coke for most of my students) with dimensions of axial length about 5 times that of the diameter. You want to drink it cold (at a reasonably less than room temperature). If you are given the choice of putting the bottle inside a refrigerator to cool it, how (on Earth) will you place the bottle full of drink so that it cools faster: horizontally or vertically (see Fig. 1)?

Read on to know the science behind your correct answer.

Treating the bottle full of drink as a thermodynamic closed system (one which doesn't allow Mass to escape out or get in, but allows the same with Energy) one can readily see, if the bottle needs to be cooled, it needs to release its energy out to the environment (refrigerator in this case, which is assumed to be at a lesser temperature) so that it cools (temperature, the representation of how much internal energy there is in the bottle, is lowered). And the objective is to do this as quickly as possible.

Writing First Law of Thermodynamics for the closed *bottle* system

$$\frac{dE}{dt} = -Q_{drink \rightarrow air} \quad (1)$$

where E is the internal energy of the bottle of drink and the equation indicates per second how much of that is expelled out as heat Q into the surrounding (refrigerated) air. Since we don't have a meter that measures directly this

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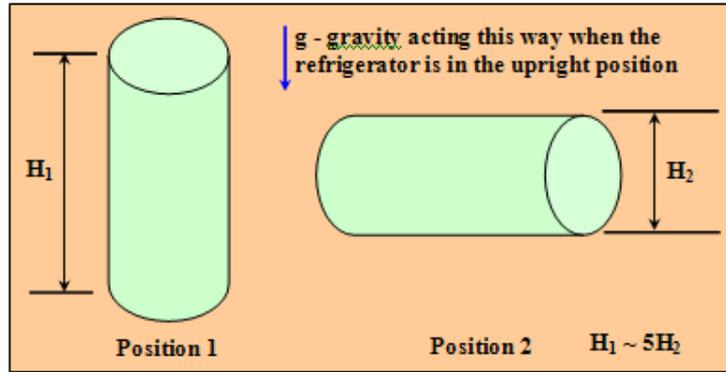


Fig. 1. Which position would cool the liquid quicker?

E and Q , we resort to simplifying Eq. (1) into some measurable quantities, leading us to

$$(mc)_{drink} \cdot \frac{dT_{drink}}{dt} = -hA(T_{drink} - T_{air}) \quad (2)$$

where mc is the thermal capacity of the drink in the bottle, h the convection heat transfer coefficient and A is the surface area of the bottle across which the convection of heat from the drink to the air takes place. That means, A is the bottle surface area "wetted" by the surrounding air on the outside and by the drink on the inside (assuming negligible thickness for the bottle wall).

To find the time required to cool the drink from its initial highest temperature T_{drink} to any lower temperature, one could rearrange suitably Eq. (2) into a 'variable separable' form and simply integrate the resulting equation to arrive at the necessary solution. Doing so, would result in an equation similar to

$$t \sim \frac{(mc)_{drink}}{hA} \quad (3)$$

Of course, one can arrive to Eq. (3) from Eq. (2) by scaling arguments, if one recognizes that the time it takes for the drink to cool from its original temperature T_{drink} to the final possible temperature of T_{air} is the maximum possible one, which can be found as Eq. (3), by setting the dT_{drink} as equal to the $(T_{drink} - T_{air})$ in Eq. (2).

An interesting thing to observe from Eq. (3) is that except for the 'h' the rest of the quantities (such as m , c and A) can be safely assumed not to vary with the orientation of the drink bottle (will this assumption ever fail?). This means the time taken for the drink in the bottle to cool is simply inversely proportional to the convection heat transfer coefficient - higher the 'h', the quicker the drink cools.

Several ways of enhancing the h exist and can be basically differentiated through what type of convection situation one ends up with namely, natural or forced (free or paid). For instance, in our situation, it is intuitively obvious for us if we blow over the bottle (forced convection), we could cool it faster than when it is required to cool by itself (natural convection).

Unfortunately, we don't expect ourselves to sit inside our fridge and keep blowing over the bottle, so it is all right to assume that the bottle be allowed to cool by natural convection mode of heat transfer.

Since we are given the choice of two positions (see Fig. 1), it is safe to assume that the natural convection heat transfer coefficient would depend on the position of the bottle (else we have no problem at hand). Therefore one can write for the two positions given in Fig. 1

$$\frac{t_1}{t_2} = \frac{h_2}{h_1} \quad (4)$$

All that now remains is to estimate the overall heat transfer coefficient by some means. In principle the overall h is a combination of the heat transfer coefficients of both the drink and the air, as both offer resistance for the energy to get transferred as heat from the drink to the air. For the sake of brevity of this post, let us take it for granted that

$$h_{drink} \gg h_{air} \quad (5)$$

(This is true anyway for most of the drinks including coke and beer [1]. For water, h_{water} is about 60 times larger than h_{air} at $10^\circ C$).

In other words, the overall h in Eq. (4) is controlled more by h_{air} and so if one finds that out, it can be safely substituted into Eq. (4) to find the answer for the cooling time.

Now from research carried out so far and well documented in text books of convection heat transfer [1], one can learn that the natural convection heat transfer coefficient h depends on the Rayleigh Number of the configuration in the following way

$$h \sim h_{air} \sim \frac{1}{H} \cdot (kRa_H^{1/4})_{air} = (\text{constant}) \cdot H^{-1/4} \quad (6)$$

(k is the thermal conductivity of air, which is treated as a constant). For a given temperature difference between the drink and the air, and known thermo-physical properties of air (density, kinematic viscosity, coefficient of

thermal expansion), this means, the h simply is inversely proportional to the height H of the configuration (the drink bottle, in our case) that is undergoing the natural convection cooling.

Using this Eq. (6) into Eq. (4) along with the fact that H_1 is five times that of H_2 (as given in Fig. 1), it is obvious that

$$\frac{t_1}{t_2} = \frac{h_2}{h_1} = \left(\frac{H_1}{H_2}\right)^{1/4} = 5^{1/4} = 1.50 \quad (7)$$

This means the time required to cool in the vertical position (position 1 in Fig. 1) for the drink bottle is 50 % *longer* than that required when the bottle is kept in horizontal position (position 2 in Fig. 2) inside the refrigerator.

Chill that drink quickly by placing it horizontally inside the fridge.

Now there are a host of interesting things that remain to be discussed, which of course is left for future posts. If you don't believe me or the equations here, just take two identical bottles of water and put one in the door-side vertical slot and the other on the horizontal tray in a standard fridge and do the experiment for your self. If you can find two small bottles that can fit into the freezer cabinet upright (vertically), you can reach this cool result even quicker.

Reference

[1] Convection Heat Transfer by A. Bejan, in which the above situation is given as an end of the chapter exercise.